Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 26 (2020) 422–429

Structural Integrity Procedia

www.elsevier.com/locate/procedia

The 1st Mediterranean Conference on Fracture and Structural Integrity, MedFract1

A new simple method for shell vibration analysis with initial stress accounting

Dubyk Yaroslav[a]*, Ishchenko Oleksii[a,b], Kryshchuk Mykola[b]

[a]*LLC "IPP-Centre", Kyiv 01014, Ukraine*
[b]*NTUU «I. Sikorsky KPI», Kyiv 03056, Ukraine*

Abstract

This paper presents a simple semi-analytical approach for free vibration of cylindrical shell with initial prestress based on equivalent load method and the Donell-Mushtari theory. In most practical applications, shells are subjected to static loadings causing internal stress field. The presence of such initial forces like internal pressure, axial force, centripetal force and torque moment significantly affects the natural frequency spectra. According to Calladin's equivalent load method initial stress field create additional curvatures and can be added as additional terms to the basic equations. The results of presented method agree well with experimental data found in the literature. Effects of elastic support stiffness, the shell length and radius to thickness ration on natural frequencies are investigated.

Keywords: Cylindrical shells; initial stress;

1. Introduction

Cylindrical shells are the most studied type of shell and their behavior describes many theories and solutions. The approximate solutions of shells are presented in papers Matsunaga (2009), Qu et al (2013), Viola et al (2013), they are based on approximation theories and do not have high accuracy. Other solutions applied by Xing et al (2013), Tong et

* Corresponding author. Tel.:+38-044-502-45-70.
E-mail address: dubykir@gmail.com

10.1016/j.prostr.2020.06.055

al (2018) based on exact theories do not work for all types of boundary conditions. In most practical applications, shells are subjected to static loadings causing internal stress field. For example, reactor pressure vessel and nuclear piping experiencing significant stress due to internal pressure. Thus, it would be practically valuable to have unite solution that could be applied both for unloaded and prestressed shell. The effect of internal pressure on a cylindrical shell was considered using approximating theory of Love by Kandasamy et al (2016). Study of Isvandzibaei et al (2013) was conducted with first order shear deformation theory for cylindrical shells with ring support under internal pressure. The solutions for fluid-filled cylindrical shells are discussed for example in Vamsi Krishna and Ganesan (2006) and Daud and Viswanathan (2019). Shells under arbitrary boundary conditions and with varied initial stresses in different longitudinal sections are analyzed by Li et al (2011). All these solutions are very specialized, do not cover all types of boundary conditions and all possible stress field. Thus, it would be valuable to have a simple and versatile engineering solution for prestressed cylindrical shell vibrations, which will cover all types of boundary conditions including elastic supports.

The studies were conducted using the equivalent load method, which was proposed by Calladine (1972) for study shells with shape imperfections like geometrical defects. Method essence lies in the fact that these imperfections create additional curvatures on which additional loads arise. Calladine (1972) proposed to consider the stress state of shells as a superposition of two states: an ideal shell with external loads and an ideal shell with an equivalent system of loading arising from shape imperfections:

$$p \approx \tilde{N}_x \chi_{xx} + 2\tilde{N}_{x\varphi}\, \chi_{x\varphi} + \tilde{N}_\varphi \chi_{\varphi\varphi} \tag{1}$$

This method has proved its practical application for small geometry imperfections, thus goal of this work is to expand it to the study of vibrations of shells with initial stresses. This work concentrates on study the most important uniform prestress (not varying with the spatial coordinates, x and φ). These loads can occur, for example, for pressurized (internal or external) cylinders, or for shells spinning about their longitudinal axes.

Nomenclature

R, h, L	mean radius, shell thickness and length
E, μ, ρ	Young's modulus, Poisson ratio and density of shell material
ω	frequency
N_x, N_φ	axial and circumferential normal forces
$N_{x\varphi}$	shear force
Q_x, Q_ϕ	axial and circumferential bending forces
u, v, w	axial, circumferential and radial displacements
m, n	wave number in circumferential and axial direction
$\chi_{xx},\ \chi_{\varphi\varphi}$	bending strains in axial and circumferential direction
$\chi_{x\varphi}$	bending strain of torque
$\tilde{P}, \tilde{N}, \tilde{M}$	pressure, axial force and torque moment
$\tilde{N}_x,\ \tilde{N}_{x\varphi},\ \tilde{N}_\varphi$	forces in shell from initial stresses
$H = Eh/(1-\mu^2)$	shell extensional modulus.

2. Mathematical formulation

According to the Donell-Mushtari thin shell theory, the governing balance equations for free vibration analysis of a uniform circular cylindrical shell are written as:

$$\frac{\partial N_x}{\partial x}+\frac{\partial L}{R\partial\varphi}+\rho h\ddot{u}=0 \tag{2}$$

$$\frac{\partial N_\varphi}{R\partial\varphi}+\frac{\partial L}{\partial x}+\frac{Q_\varphi}{R}+\rho h\ddot{v}=0 \tag{3}$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_\varphi}{R\partial\varphi}-\frac{N_\varphi}{R}+\rho h\ddot{w}=0 \tag{4}$$

Here over dots $\ddot{u},\ddot{v},\ddot{w}$ denotes double time derivatives of displacements. The initial stresses in the shell from the internal (external) pressure $\tilde{N}_\varphi=\tilde{P}R$, axial force $\tilde{N}_x=\tilde{N}/2\pi R$, rotational speed $\tilde{N}_\varphi=\tilde{\omega}^2\rho hR^2$ and torque moment $\tilde{N}_{x\varphi}=M_{tor}/2\pi R^2$, form the following curvatures:

$$\chi_\varphi=w+\frac{\partial^2 w}{\partial\varphi^2};\ \chi_x=\frac{\partial^2 w}{\partial x^2};\ \chi_{x\varphi}=\frac{\partial v}{\partial x}-\frac{1}{R}\frac{\partial u}{\partial\varphi}-2\frac{\partial^2 w}{\partial x\partial\varphi} \tag{5}$$

We have to add these terms only in equation (4):

$$\begin{aligned}&\frac{\partial Q_x}{\partial x}+\frac{\partial Q_\phi}{R\partial\phi}-\frac{N_\phi}{R}-\left(\frac{\tilde{P}}{R}-\tilde{\omega}^2\rho h\right)\left(w+\frac{\partial^2 w}{\partial\varphi^2}\right)-\tilde{N}_x\frac{\partial^2 w}{\partial x^2}+\\&+\tilde{N}_{x\varphi}\left(\frac{\partial v}{\partial x}-\frac{1}{R}\frac{\partial u}{\partial\varphi}-2\frac{\partial^2 w}{\partial x\partial\varphi}\right)+\rho h\ddot{w}=0\end{aligned} \tag{6}$$

Using the calculation procedure given in our previous work Dubyk et al (2018a), by combining the static equations with the physical and geometric, we obtain an eighth-order differential equation. Using the expansion of eight parameters in trigonometric series:

$$\Phi=\left(\sum_{n=0}\varphi_n\cos n\varphi\ \text{ or }\ \sum_{n=1}\varphi_n\sin n\varphi\right) \tag{7}$$

We can obtain a system of eight ordinary differential equations, in which only one equation for $\frac{dq_x(x)}{dx}$ is changed. A full system of eight ordinary differential equations describing a prestressed cylindrical shell:

$$\frac{dn_x(x)}{dx}=\frac{n}{R}n_{x\varphi}(x)-\Omega^2 u(x) \tag{8}$$

$$\frac{dn_{x\varphi}(x)}{dx}=\left(\frac{\delta}{R^2}-\mu\right)n\frac{n_x(x)}{R}-\frac{n\delta}{R^3}m_x(x)+\left(\frac{n^2}{R}-\left[1-\frac{\delta}{R^2}\right]\Omega^2\right)v(x)+\left(\frac{n}{R}+\frac{\delta n\left(n^2-1\right)}{R^3(1+\mu)}\right)w(x) \tag{9}$$

$$\frac{dq_x(x)}{dx}=\left(n^2\frac{\delta}{R^2}-\mu\right)\frac{n_x(x)}{R}-\frac{n^2\delta}{R^3}m_x(x)+\left(\frac{n}{R}+\frac{\Omega^2\delta n}{R^2}\right)v(x)+\left(\frac{1}{R}-\Omega^2+\frac{\delta n^2\left(n^2-1\right)}{R^3\left(1+\mu\right)}\right)w(x)$$
$$+\left(\frac{\tilde{P}}{R}-\tilde{\omega}^2\rho h\right)\left(1-n^2\right)w(x)+\tilde{N}\left[\frac{1-\mu^2}{R}m_x(x)+\mu\frac{n}{R}v(x)+\mu\frac{n^2}{R}w(x)\right]-$$
$$-\tilde{N}_{x\varphi}\left(2\frac{1+\mu}{R}n_{x\varphi}(x)-\frac{n}{R}u(x)+\frac{n}{R}+2n\frac{1}{R}\gamma_x(x)\right) \quad (10)$$

$$\frac{dm_x(x)}{dx}=\frac{n}{R}n_{x\varphi}(x)-q_x(x)-\frac{1}{2}\frac{n^2}{(1+\mu)R}u(x)+\frac{n^2}{(1+\mu)R}\gamma_x(x) \quad (11)$$

$$\frac{du(x)}{dx}=\frac{1-\mu^2}{R}n_x(x)+\mu\frac{n}{R}v(x)+\mu\frac{1}{R}w(x) \quad (12)$$

$$\frac{dv(x)}{dx}=2\frac{1+\mu}{R}n_{x\varphi}(x)-\frac{n}{R}u(x) \quad (13)$$

$$\frac{dw(x)}{dx}=\frac{1}{R}\gamma_x(x) \quad (14)$$

$$\frac{d\gamma_x(x)}{dx}=\frac{1-\mu^2}{R}m_x(x)+\mu\frac{n}{R}v(x)+\mu\frac{n^2}{R}w(x) \quad (15)$$

Here we used notation $\Omega^2=\omega^2\frac{\rho R}{E}, \delta=\frac{h^2}{12}$

System of eq. (8)-(15) can be easily solved using expansion in ordinary polynomials for axial coordinate $x^0, x^1, x^2 \ldots$ For sake of simplicity the solution is rewritten using the method of initial parameters:

$$\begin{aligned} n_x(x)&=n_{x0}+C_{11}\cdot x+C_{12}\cdot x^2+C_{13}\cdot x^3+\ldots \\ &\ldots \\ \gamma_x(x)&=\gamma_{x0}+C_{81}\cdot x+C_{82}\cdot x^2+C_{83}\cdot x^3+\ldots \end{aligned} \quad (16)$$

It is practical to limit our solution with fourth degree polynomials and to achieve good accuracy we can just ‘slice’ our shell in axial direction, for every sliced part solution (16) is applied. At each edge of the shell four boundary conditions must be specified. They can be generalized by the following equations:

$$Q_x+\frac{\partial M_{x\varphi}}{\partial\varphi}=k_w w \quad (17)$$

$$M_x=k_\gamma\gamma_x \quad (18)$$

$$N_x = k_u u \tag{19}$$

$$N_{x\varphi} - \frac{1}{R} M_{x\varphi} = k_v v \tag{20}$$

Here $k_u, k_v, k_w, k_\gamma -$ stands for axial circumferential, radial and rotational spring stiffness, i.e. eq.(17)-(20) is a general type elastically supported edge. If we consider $k_u = k_v = k_w = k_\gamma = 0$ we will get free edge ‘F’ and opposite $k_u = k_v = k_w = k_\gamma = \infty$ is a clamped edge ‘C’. A full description of the possible boundary conditions is also presented in paper Dubyk et al (2018a).

3. Results and discussion

The proposed semi-analytical method is applied to calculate natural frequencies of circular cylindrical shells with arbitrary boundary conditions and initial prestress. But before it we have checked the convergency criteria for proposed semi-analytical method, depending on the division in axial direction (Fig.1). It can be seen that 50 divisions in axial direction are quite enough, also it is possible to limit our solution with polynomials of 3[rd] grade.

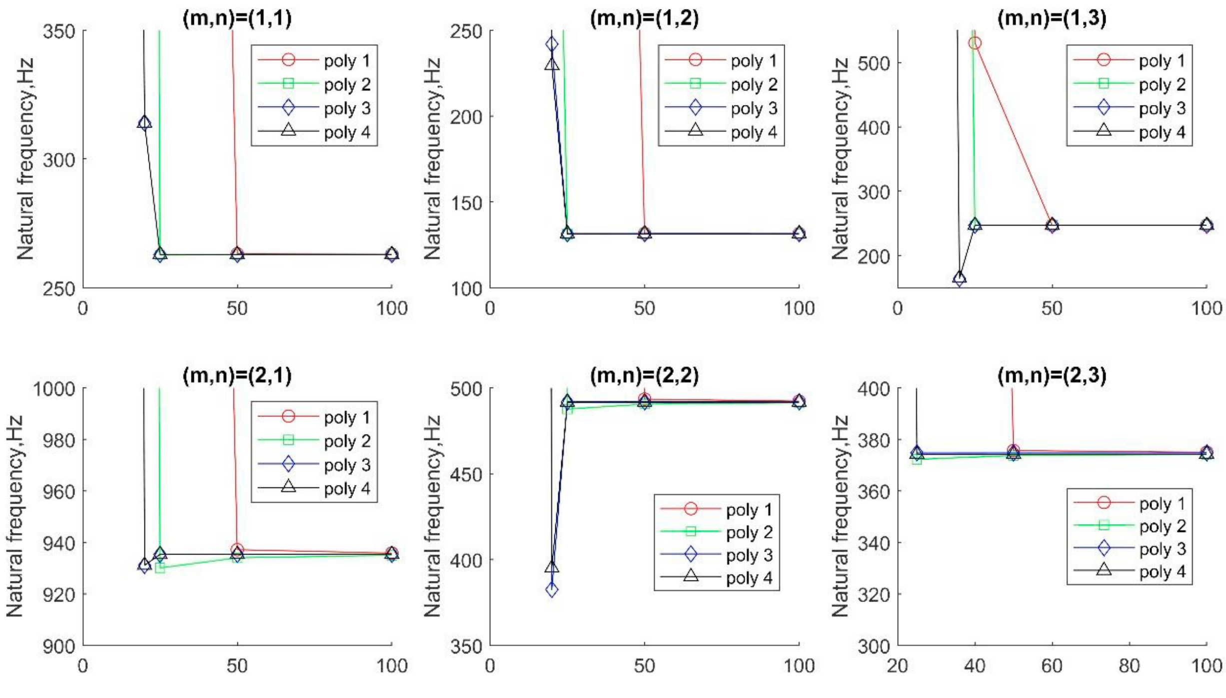

Fig. 1. Natural frequencies of the cylindrical shell versus number of divisions in axial direction: C-F, L = 1.25; R = 0.25; h = 0.008.

In Fig. 2-Fig. 3 the present method is validated by comparing with data published in literature. Comparison is made for axial and circumferential prestress, for two types of boundary conditions: simply supported and clamped. From the analysis of experimental and calculated data for the simply supported shell (see Fig. 2), it follows that the axial force significantly reduces the frequency values when the number of axial and circumferential waves is greater than unity. The same behavior can be seen in Fig 3, where the influence of circumferential and combined axial and circumferential prestress is analyzed. In Fig. 4 influence of the increasing internal pressure on natural frequencies is demonstrated. We can note that due to the supporting action of internal pressure, the frequencies of the cylindrical shell increase.

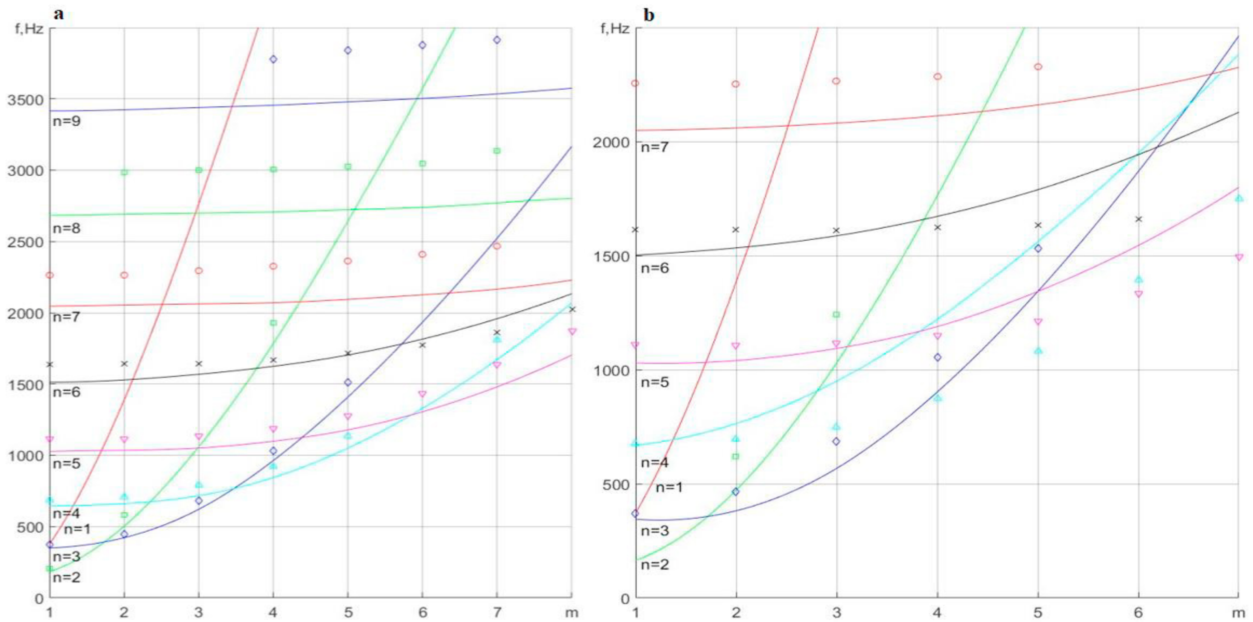

Fig. 2. Natural frequencies of the simply supported shell S-S L/R=19.33, R/h =150, μ=0.3, without loads (a) with axial force (b) experimental data obtained by Herrmann and Shaw (1965): (☐) n=2, n=8, (◊) n = 3, n=9, (Δ) n = 4, (▼) n = 5, (×) n = 6, (○) n=7, ▬▬▬ our solution.

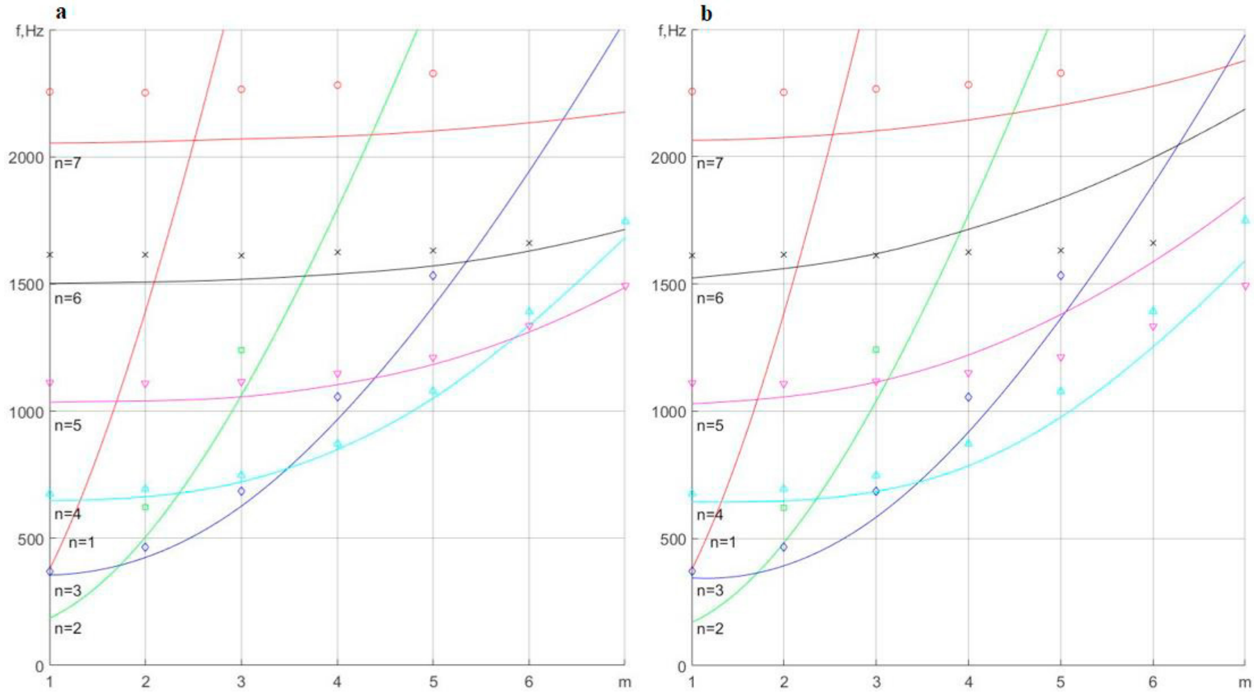

Fig. 3. Natural frequencies of the simply supported shell S-S L/R=19.33, R/h =150, μ=0.3, with internal pressure (a) with combine internal pressure and axial force (b) experimental data obtained by Herrmann and Shaw (1965) (☐) n=2, (◊) n = 3, (Δ) n = 4, (▼) n = 5, (×) n = 6, (○) n=7, ▬▬▬ our solution.

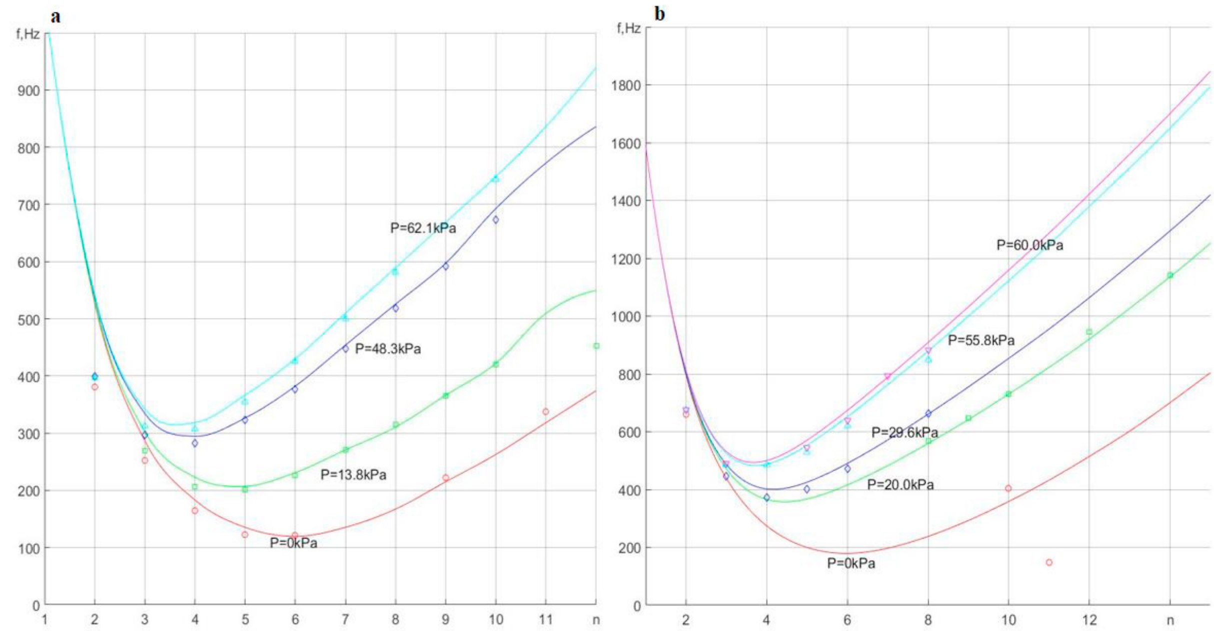

Fig. 4. Natural frequencies of the clamped supported shell C-C μ=0.32, L/R=6, R/h =601 (a) та R/h =666 (b) with internal pressure for m=1: experimental data obtained by Miserentino and Vosteen (1965) for (a): (○) P=0 kPa, (□) P=13.8 kPa, (◊) P = 48.3 kPa, (Δ) P = 62.1 kPa, for (b): (○) P=0 kPa, (□) P=20 kPa, (◊) P = 29.6 kPa, (Δ) P = 55.8 kPa, (▼) P = 60 kPa, ——— our solution.

Having gained confidence in the present method, natural frequencies of a circular cylindrical shell with elastically supported boundary conditions are calculated eq.(17)-(20). Presented in Table.1 results with no initial prestress coincide with presented by Dai et al. (2013).

Table 1. Natural frequencies of clamped-elastically supported shell: l=1.25m, R=0.25m, h=0.008m, E=210GPa, ρ=7800kg/m3, μ=0.3, ku=kv=kγ=0

Mode	k_w / H=0	k_w / H=0.01	k_w / H=0.1	k_w / H=1	k_w / H=1e6	k_w / H=1e8
			No initial prestress			
1	131.6	183.4	299.1	316.0	316.6	316.6
2	247.0	278.2	310.8	340.7	345.9	345.9
3	262.9	279.9	365.5	476.1	492.1	492.1
4	374.8	402.9	490.8	492.0	505.4	505.4
			Axial prestress $\tilde{N}_x = 0.001H$			
1	124.7	187.8	312.6	327.1	327.7	327.7
2	235.7	280.4	321.6	351.8	356.5	356.5
3	262.9	282.7	378.8	494.4	499.6	499.6
4	382.5	412.6	498.0	499.4	523.2	523.2
			Circumferential prestress $\tilde{N}_\varphi = 0.001H$			
1	214.3	249.5	343.5	380.1	384.7	384.7
2	262.9	279.9	365.4	430.8	431.2	431.2
3	383.7	404.4	427.1	476.1	505.3	505.3
4	474.9	497.4	603.5	640.9	641.0	641.0

Results presented in Fig.2-4 and Table 1 show that, the proposed semi-analytical method agree well with those data presented in the published literature. Thus, the present method can be used for high accuracy modeling of forced vibration or dynamic analysis of loaded constructions (for example see analysis of water hammer event in Dubyk et al. (2018b)), that can be schematized as cylindrical shells.

4. Conclusions

In this work an accurate semi-analytical solution of free vibration frequencies of prestressed cylindrical shell, based on the Donell-Mushtari theory, is obtained using polynomials expansion in axial directions and Fourier series in circumferential direction:

- Eight main variables are selected, they are used to write out all the equations and boundary conditions. This formulation allowed us to solve a system of partial differential equations using series expansion. Also, this formulation is suitable to address elastically supported edges, which are generalization of classical boundary conditions.
- Our solution is versatile and can be easily extended to account for initial stresses like axial force, pressure (internal and external), torque moment and centripetal force. We just need to adjust third balance equation for initial prestress. These results were checked against experimental data and good convergency for low frequency spectra is obtained.

References

Matsunaga, H., 2009. Free vibration and stability of functionally graded circular cylindrical shells according to a 2D higher-order deformation theory. Coposite Structures 88, 519–531.

Qu, Y., Hua, H., Meng, G., 2013. A domain decomposition approach for vibration analysis of isotropic and composite cylindrical shells with arbitrary boundaries. Composite Structures 95, 307–321.

Viola, E., Tornabene, F., Fantuzzi, N., 2013. General higher-order shear deformation theories for the free vibration analysis of completely doubly-curved laminated shells and panels. Composite Structures 95, 639–666.

Xing, Y., Liu, B., Xu, T., 2013. Exact solutions for free vibration of circular cylindrical shells with classical boundary conditions. International Journal of Mechanics Sciences 75, 178–188.

Tong, Z., Ni, Y., Zhou, Z., Xu, X., Zhu, S., Miao, X., 2018 Exact Solutions for free vibration of cylindrical shells by a symplectic approach. Journal of Vibration Engineering & Technologies 6, 107–115.

Kandasamy, J., Madhavi, M., Haritha, N., 2016. Free vibration analysis of thin cylindrical shells subjected to internal pressure and finite element analysis. International Journal of Research in Engineering and Technology 5, 40–48.

Isvandzibaei, M., Jamaluddin, H., Hamzah, R., 2013. Effects of ring support and internal pressure on the vibration behavior of multiple layered cylindrical shells. Advances in Mechanical Engineering, 1–13.

Vamsi Krishna, B., Ganesan, N., 2006. Polynomial approach for calculating added mass for fluid-filled cylindrical shells. Journal of sound and vibration 291, 1221–1228.

Daud, N., Viswanathan, K., 2019. Vibration of symmetrically layered angle-ply cylindrical shells filled with fluid. Plos One, 1–18.

Li, D., Lei, Y., 2011. Free vibration of a cylindrical shell with varied initial stresses in different longitudinal sections. Applied Mechanics and Materials Vols. 52-54, 717–722.

Calladine, C., 1972. Structural consequences of small imperfections in elastic thin shells of revolution. Int. J. Solids Structures 8, 679–697.

Dubyk, Y., Orynyak, I., Ishchenko, O., 2018a. An exact series solution for free vibration of cylindrical shell with arbitrary boundary conditions. Scientific Journal of the Ternopil National Technical University 1, 79–89.

Herrmann, G., Shaw J., 1965. Vibration of thin shells under initial stress. Journal Eng. Mech. Division 91, 37–59.

Miserentino, R., Vosteen, L., 1965. Vibration tests of pressurized thin-walled cylindrical shells. in *"National Aeronautics and Space Administration,* Washington, p. 50.

Dai L., Yang T., Du J., Li. W., Brennan M., 2013 An exact series solution for the vibration analysis of cylindrical shells with arbitrary boundary conditions, Applied Acoustics 74, 440-449.

Dubyk, Y., Filonov V., Ishchenko O., Orynyak I., Filonova Y. 2018b. Dynamic assessment of the core barrel during loss of coolant accident. Proceedings of the ASME 2018 Pressure Vessels and Piping Conference PVP2018-84762, Czech Republic, July, 15-20, 2018, p.10.